

Find the vertex, focus, directrix, axis of symmetry, and direction (up, down, right, or left) for the following parabola:

$$y^2 - 12x - 2y + 25 = 0$$

We need to get this equation in the standard form for a parabola.

- 1) Which variable is being squared (x or y)?

y

- 2) Moved the squared variable stuff to the right

Since y is being squared, we will move the y^2 & y terms to the right.

$$\begin{aligned} y^2 - 12x - 2y + 25 &= 0 \\ -12x + 25 &= -y^2 + 2y \end{aligned}$$

- 3) Complete the square on the right (and balance on the left)

$$\begin{aligned} -12x + 25 &= -y^2 + 2y \\ -12x + 25 &= -1(y^2 - 2y \quad) \end{aligned}$$

$$\begin{aligned} -12x + 25 + (-1) &= -1(y^2 - 2y + 1) \\ -12x + 24 &= -1(y - 1)^2 \end{aligned}$$

[We are getting close! The right side looks like a parabola equation]

- 4) Factor left side to make coefficient = 1

$$\begin{aligned} -12x + 24 &= -1(y - 1)^2 \\ -12(x - 2) &= -1(y - 1)^2 \end{aligned}$$

- 5) Multiply both sides by reciprocal of number you factored in Step 4

$$-12(x - 2) = -1(y - 1)^2$$

We will multiply by $-\frac{1}{12}$ on both sides.

$$-\frac{1}{12}[-12(x - 2)] = -\frac{1}{12}[-1(y - 1)^2]$$

$x - 2 = \frac{1}{12}(y - 1)^2$ is the standard equation for our parabola.

Now we can find the vertex, focus, directrix, axis, and direction!

$$x - 2 = \frac{1}{12}(y - 1)^2$$

Vertex

We can see that the **vertex** has to be $(2, 1)$.

Direction

There are 4 choices:

UP DOWN RIGHT LEFT

Look at the equation – which is being squared – x or y ?

Since y is being squared, we eliminate two choices and keep two possibilities...

~~UP~~ ~~DOWN~~ RIGHT LEFT

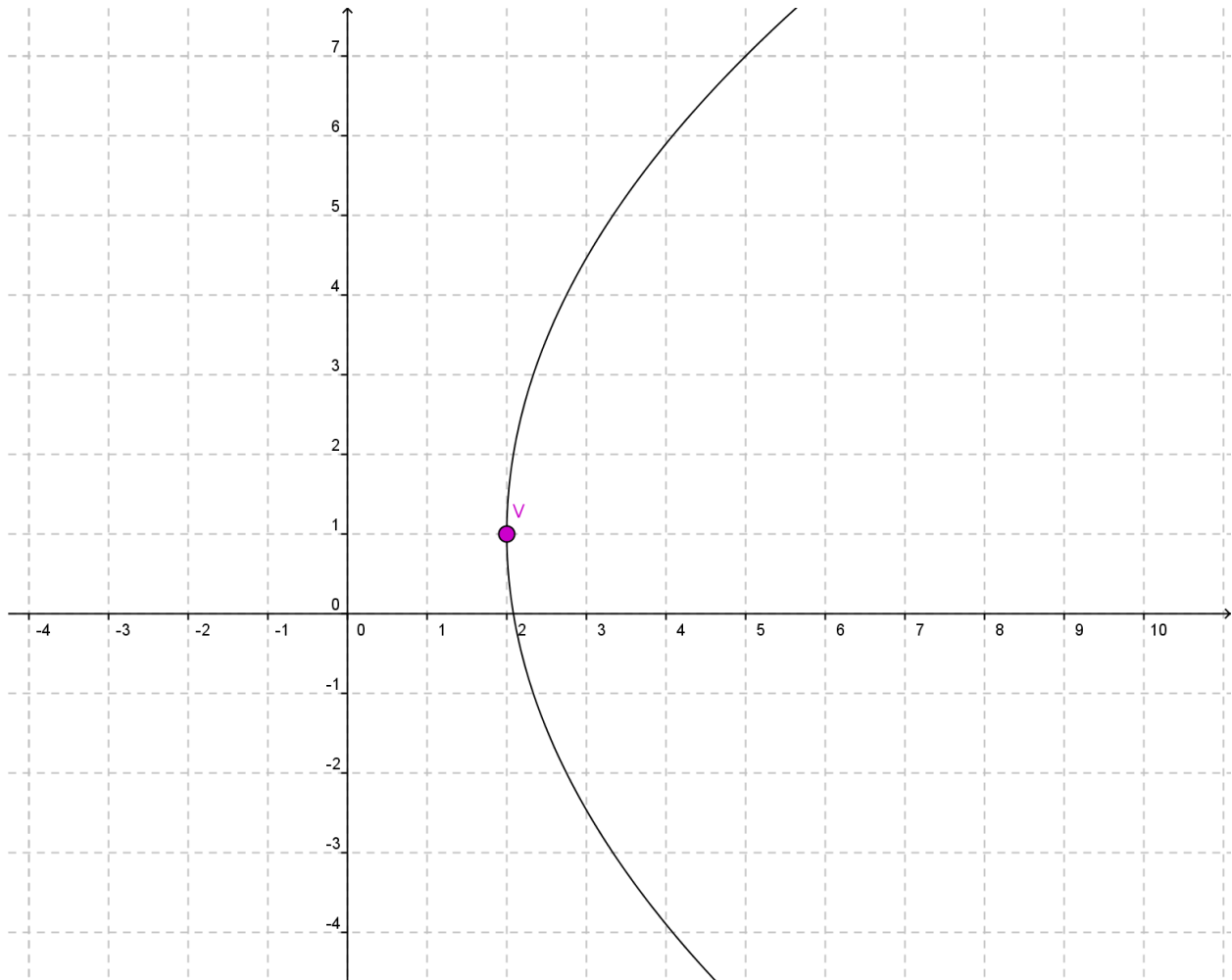
Now look at a – is a positive or negative?

Since a is positive, we know which direction to keep...

~~UP~~ ~~DOWN~~ **RIGHT** ~~LEFT~~

Sketch a graph

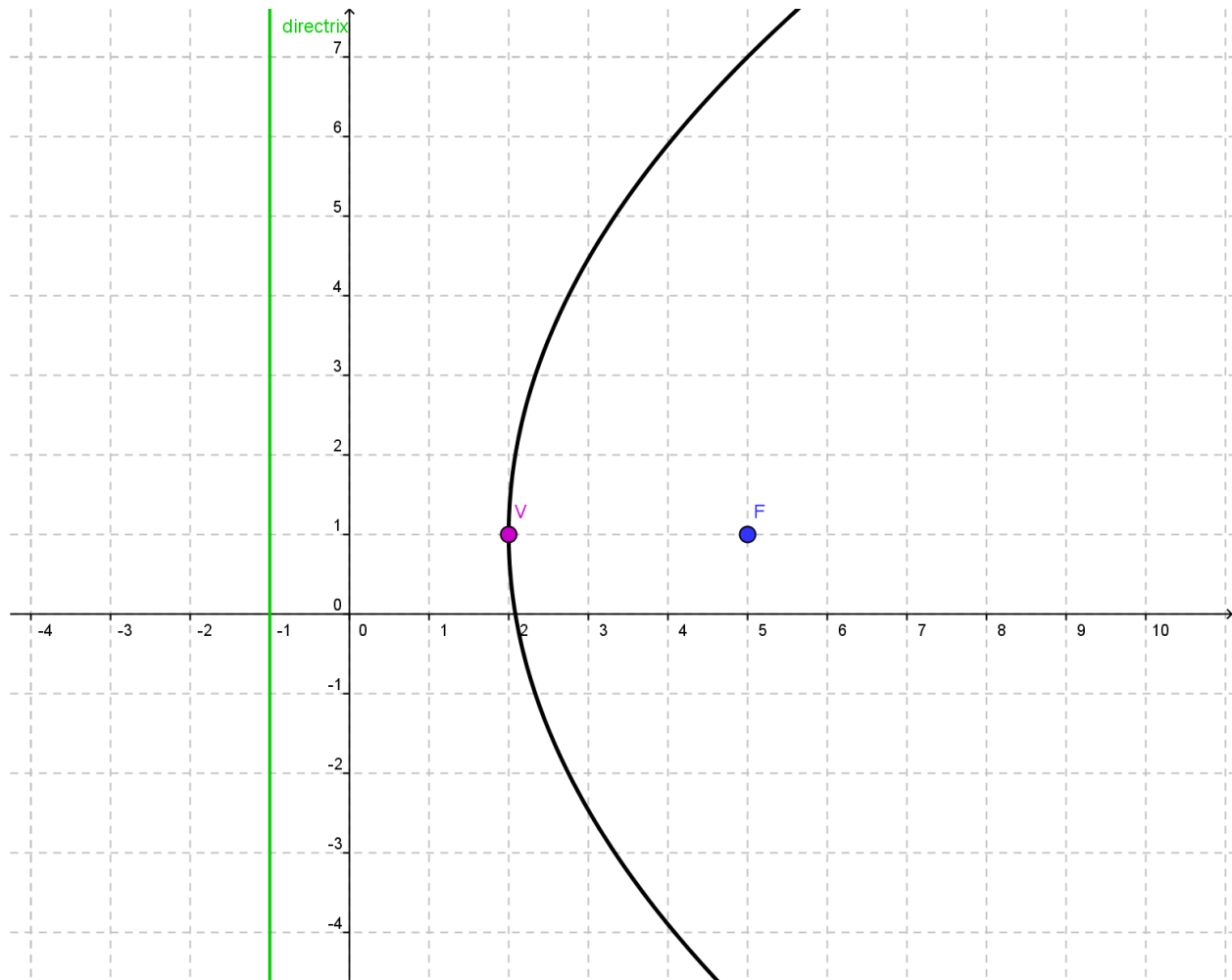
This is a good point to sketch a parabola that opens to the RIGHT with a **vertex** $(2, 1)$. Don't worry about the accuracy of the width of your sketch- the correct direction and vertex are good enough!

Determine the value of c

Remember that $a = \frac{1}{4c}$. Since $a = \frac{1}{12}$ in our equation,

$$\frac{1}{4c} = \frac{1}{12} \longrightarrow 4c = 12 \longrightarrow c = 3$$

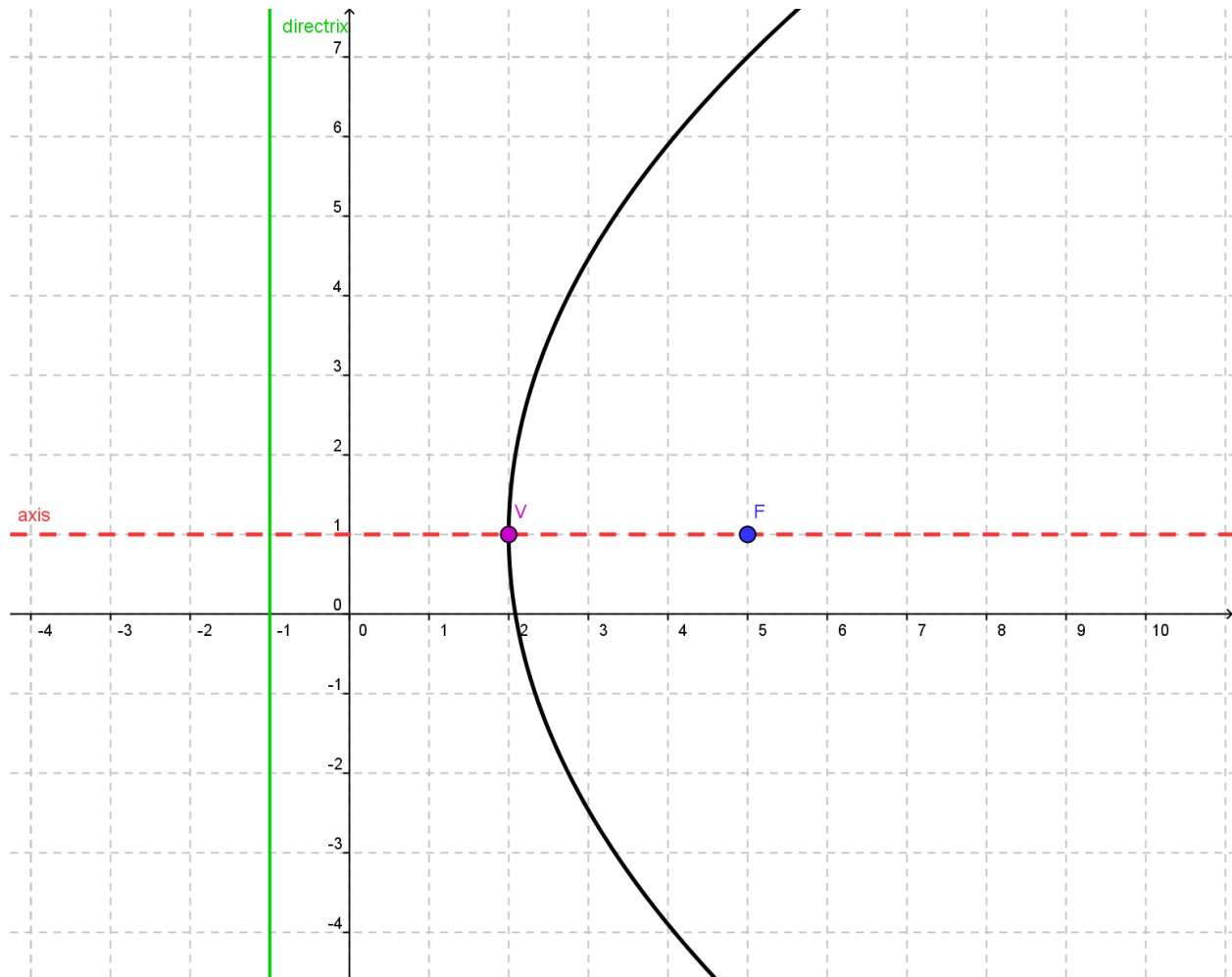
The **focus** is 3 units to the right (same direction the parabola opens) of the vertex. The **directrix** is 3 units to the left (opposite direction the parabola opens) of the vertex.

Graph the focus and directrix

The **focus** is the point $(5, 1)$ and the **directrix** is the line $x = -1$.

Draw the axis of symmetry

The **axis of symmetry** is a line through the **vertex** and the **focus**.



The **axis of symmetry** is the line $y = 1$.